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# Broadband left-handed materials made of thin soft ferromagnetic films with in-plane uniaxial anisotropy

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# Abstract

We study theoretically characteristics of normally incident E-polarized microwave propagation in soft ferromagnetic films with in-plane uniaxial anisotropy, and find that due to the existence of the gyrotropic permeability tensor, the soft ferromagnetic film may become a left-handed material over a broad frequency band. Furthermore, the bandwidth and the frequency positions can be easily modified by the external magnetic field. Finally, the energy loss of the microwave propagating in the film is investigated.

(Some figures in this article are in colour only in the electronic version)

## 1. Introduction

In 1960s, Veselago [1] theoretically predicted the existence of left-handed material (LHM): the permittivity  $\varepsilon$  and permeability  $\mu$  of the material are negative simultaneously; the electric field  $\vec{E}$ , magnetic field  $\vec{H}$  and wavevector  $\vec{k}$  of a electromagnetic wave propagating in it form a left-handed triplet of vectors. On the basis of the early ideas of Pendry *et al* [2, 3], Smith *et al* [4] found negative effective  $\mu_{\text{eff}}$  and  $\varepsilon_{\text{eff}}$  simultaneously in a system consisting of spilt ring resonators and metal wires, and Shelby *et al* [5] observed negative values of the refraction index *n* in this system. This discovery reinvigorated the research for LHM [6–19]. Various interesting properties of LHM were discussed, such as negative refraction [6], imaging of superlens [7], enhancement of evanescent waves [7, 8].

Thin soft ferromagnetic metallic films have attracted much attention in view of the wide potential applications in data storage, magnetic sensors and microwave devices [9–11, 20]. It is well known that at the ferromagnetic resonance with right or linearly polarized radiation there is a frequency range where the real part of the permeability is negative. In addition, according to the Drude model, the real part of the permittivity of the metallic film is negative below the plasma frequency. Therefore, metallic ferromagnetic films, such as metallic magnetic granular



Figure 1. The studied metallic ferromagnetic film. The inset shows the kDB  $(0\eta\xi\zeta)$  coordinate system used in the calculation of wave eigenmodes.

composites [9], metallic magnetic thin films [10], coupled ferromagnetic bilayer films [11], were proposed as LHM in a certain frequency range. It is noted that the previous works have mainly focused on the characteristics of the microwave propagation in the direction of the magnetization. Since the thin soft ferromagnetic metallic films usually have in-plane uniaxial anisotropy [20], and the direction of normally incident microwaves is perpendicular to the magnetization, it is also interesting to study the properties of microwave propagation in the direction perpendicular to the magnetization.

This paper is organized as follows. In section 2, we give the theories of the normally incident microwave propagation in the soft ferromagnetic film with in-plane uniaxial anisotropy. In section 3, on the basis of the obtained equations, we numerically study the characteristics of the normally incident E-polarized microwave propagation in the films. Finally, some conclusions are demonstrated in section 4.

## 2. Theories

Figure 1 illustrates a thin soft ferromagnetic film and the coordinate systems used in this work. Assuming the magnetization of the film is in the plane, defined as the *z* axis, and the film is infinite along the *y* and *z* axes, the relative permeability tensor of the thin ferromagnetic film is given by [21]

$$\mu = \begin{bmatrix} \mu_{xx} & -i\mu_{xy} & 0\\ i\mu_{yx} & \mu_{yy} & 0\\ 0 & 0 & 1 \end{bmatrix},$$
(1)

where  $\mu_{xx} = 1 + \frac{\omega_y \omega_M}{\omega_x \omega_y - \omega^2}$ ,  $\mu_{yy} = 1 + \frac{\omega_x \omega_M}{\omega_x \omega_y - \omega^2}$ ,  $\mu_{xy} = \mu_{yx} = \frac{-\omega_M \omega}{\omega_x \omega_y - \omega^2}$ ,  $\omega_M = \gamma M_s$ ,  $\omega_x = \gamma (H_0 + H_{ex} + M_s + i\alpha\omega)$ ,  $\omega_y = \gamma (H_0 + H_{ex} + i\alpha\omega)$ ,  $\gamma$  is the gyromagnetic ratio,  $M_s$  is the saturation magnetization,  $H_0$  is the anisotropy field,  $H_{ex}$  is the external field with the direction of the *z* axis, and  $\alpha$  is the ferromagnetic damping coefficient. The relative permittivity



Figure 2. Schematic diagram of a beam of waves passing from free space into the film.

of the film is also a tensor expressed as [22]

$$\varepsilon = \begin{bmatrix} \varepsilon_{xx} & -i\varepsilon_{xy} & 0\\ i\varepsilon_{yx} & \varepsilon_{yy} & 0\\ 0 & 0 & \varepsilon_{zz} \end{bmatrix},$$
(2)

where the elements of  $\varepsilon_{xx}$ ,  $\varepsilon_{yy}$  and  $\varepsilon_{xy}$  (= $\varepsilon_{yx}$ ) are related to the magnetic flux density  $\vec{B}$  and the depolarization factor of the film,  $\varepsilon_{zz} = 1 - \frac{\omega_p^2}{\omega(\omega - i/\tau)}$ ,  $\omega_p = (\frac{ne^2}{\varepsilon_0 m^*})^{1/2}$  is the plasma frequency and  $\tau$  is a relaxation time. Such gyrotropic tensors may dramatically influence the propagation properties of the polarization components [12–14].

For simplicity, we investigate the characteristics of the normally incident microwave propagation in the film as shown in figures 1 and 2. The microwave eigenmodes in the film may be expediently obtained by applying the kDB  $(0\eta\xi\zeta)$  coordinate system demonstrated in the inset of figure 1 [22]. In the calculation, the inversion relative permeability  $\nu$  and inversion relative permittivity  $\kappa$  shown below are used:

$$\nu = \begin{bmatrix} \nu_1 & -i\nu_2 & 0\\ i\nu_2 & \nu'_1 & 0\\ 0 & 0 & \nu_3 \end{bmatrix} \equiv (\mu)^{-1}, \qquad \kappa = \begin{bmatrix} \kappa_1 & -i\kappa_2 & 0\\ i\kappa_2 & \kappa'_1 & 0\\ 0 & 0 & \kappa_3 \end{bmatrix} \equiv (\varepsilon)^{-1}, \quad (3)$$

where  $\nu_1 = \frac{\mu_{yy}}{\mu_{xx}\mu_{yy}-\mu_{xy}^2}$ ,  $\nu'_1 = \frac{\mu_{xx}}{\mu_{xx}\mu_{yy}-\mu_{xy}^2}$ ,  $\nu_2 = \frac{-\mu_{xy}}{\mu_{xx}\mu_{yy}-\mu_{xy}^2}$ ,  $\nu_3 = 1.0$ ,  $\kappa_1 = \frac{\varepsilon_{yy}}{\varepsilon_{xx}\varepsilon_{yy}-\varepsilon_{xy}^2}$ ,  $\kappa'_1 = \frac{\varepsilon_{xx}}{\varepsilon_{xx}\varepsilon_{yy}-\varepsilon_{xy}^2}$ ,  $\kappa_2 = \frac{-\varepsilon_{xy}}{\varepsilon_{xx}\varepsilon_{yy}-\varepsilon_{xy}^2}$ , and  $\kappa_3 = \frac{1}{\varepsilon_{zz}}$ . Following a standard procedure we get the equations [22]

$$\begin{bmatrix} \kappa_1' & 0\\ 0 & \kappa_3 \end{bmatrix} \begin{bmatrix} D_\eta\\ D_\xi \end{bmatrix} = \begin{bmatrix} 0 & \omega \varepsilon_0 / k\\ -\omega \varepsilon_0 / k & 0 \end{bmatrix} \begin{bmatrix} B_\eta\\ B_\xi \end{bmatrix},$$
(4)

$$\begin{bmatrix} v_1' & 0\\ 0 & v_3 \end{bmatrix} \begin{bmatrix} B_\eta\\ B_\xi \end{bmatrix} = \begin{bmatrix} 0 & -\omega\mu_0/k\\ \omega\mu_0/k & 0 \end{bmatrix} \begin{bmatrix} D_\eta\\ D_\xi \end{bmatrix}$$
(5)

where k = k' - jk'' is the propagation constant in the direction parallel to the  $x(\zeta)$  axis,  $D_{\eta(\xi)}$  the electric displacement vector in the  $\eta(\xi)$  axis direction and  $B_{\eta(\xi)}$  the magnetic flux density in the  $\eta(\xi)$  axis direction, respectively. Solving equations (4) and (5), we get two types of microwave eigenmodes

(i) 
$$\kappa_{3}v_{1}' = (\omega/c_{0}k)^{2}$$
,  $\vec{D} = D\vec{e}_{\xi} = D\vec{e}_{z}$ ,  $\vec{E} = \frac{1}{\varepsilon_{0}}\kappa_{3}D\vec{e}_{z}$   
 $\vec{B} = B\vec{e}_{\eta} = B\vec{e}_{y}$ ,  $\vec{H} = -i\frac{1}{\mu_{0}}v_{2}B\vec{e}_{x} + \frac{1}{\mu_{0}}v_{1}'B\vec{e}_{y}$ ,  
(ii)  $\kappa_{1}'v_{3} = (\omega/c_{0}k)^{2}$ ,  $\vec{D} = D\vec{e}_{\eta} = D\vec{e}_{y}$ ,  $\vec{E} = -i\frac{1}{\varepsilon_{0}}\kappa_{2}D\vec{e}_{x} + \frac{1}{\varepsilon_{0}}\kappa_{1}'D\vec{e}_{y}$   
 $\vec{B} = B\vec{e}_{\xi} = B\vec{e}_{z}$ ,  $\vec{H} = \frac{1}{\mu_{0}}v_{3}B\vec{e}_{z}$ ,

where  $c_0$  is the light velocity in the free space,  $\vec{e}_x$ ,  $\vec{e}_{y(\eta)}$  and  $\vec{e}_{z(\xi)}$  are unit vectors along the  $x, y(\eta)$  and  $z(\xi)$  axes, respectively. It can be easily seen that microwave eigenmode (i) is the *E*-polarized plane microwave satisfying  $\vec{k} \cdot \vec{E} = 0$ , and microwave eigenmode (ii) is the *H*-polarized plane microwave satisfying  $\vec{k} \cdot \vec{H} = 0$ .

Let us consider a beam of microwave passing from free space into a film as shown in figures 1 and 2. To study the influence of the gyrotropic permeability tensor, we restrict ourselves to the case where the microwave propagating in the film is E-polarized, i.e., the microwave eigenmode in the film is described by solution (i). The electric fields and magnetic fields of the incident, reflected and refracted microwaves at the upper interface between the free space and the film can be expressed as

$$\vec{E}_i = E_{i0} \exp(i\omega t - ik_1 x)\vec{e}_z, \tag{6}$$

$$\vec{H}_i = \frac{-k_1}{\omega\mu_0} E_{i0} \exp(i\omega t - ik_1 x)\vec{e}_y,\tag{7}$$

$$\vec{E}_r = E_{r0} \exp(i\omega t + ik_1 x)\vec{e}_z,\tag{8}$$

$$\vec{H}_r = \frac{k_1}{\omega\mu_0} E_{r0} \exp(i\omega t + ik_1 x)\vec{e}_y.$$
(9)

$$\vec{E}_t = E_{0t} \exp(i\omega t - ikx)\vec{e}_z, \tag{10}$$

$$\vec{H}_t = \left[ -i\nu_2 \frac{-k}{\omega\mu_0} E_{0t} \vec{e}_x + \nu_1' \frac{-k}{\omega\mu_0} E_{0t} \vec{e}_y \right] \exp(i\omega t - ikx), \tag{11}$$

where  $k_1 = \omega (\mu_0 \varepsilon_0)^{1/2} = \frac{\omega}{c_0}$  is the propagation constant in the free space, and  $k = k' - jk'' = \pm \frac{\omega}{c_0 \sqrt{\kappa_3 v_1'}}$  is the propagation constant in the film; the selection of the sign  $\pm$  is to ensure that k'' is positive, i.e., the direction of the energy flow is in the positive  $x(\zeta)$  axis direction approximately [15, 16].

It is seen from equation (11) that, for the inversion relative permittivity tensor, only the element  $\kappa_3 = 1/\varepsilon_{zz}$  needs to be considered.

According to the boundary conditions

$$E_i + E_r = E_t$$

$$H_i + H_r = H_{t(y)},$$
(12)

the reflection and refraction coefficients are governed by

$$r_E = \frac{-\sqrt{\nu_1'/\kappa_3} + 1}{\sqrt{\nu_1'/\kappa_3} + 1} = \frac{-\frac{1}{\eta_s} + 1}{\frac{1}{\eta_s} + 1} = \frac{\eta_s - 1}{\eta_s + 1}$$
(13)

$$r_H = -r_E = -\frac{\eta_s - 1}{\eta_s + 1}$$
(14)

$$t_E = 1 + r_E = \frac{2\eta_s}{\eta_s + 1}$$
(15)

$$t_H = \frac{1}{\eta_s} t_E = \frac{2}{\eta_s + 1}$$
(16)

where  $\eta_{\rm s} = \sqrt{\kappa_3/\nu_1'}$  is the microwave impedance.

Furthermore, the energy current density  $\vec{S}$  and its inner product with microwave vector  $\operatorname{Re}(\vec{k}) \cdot \vec{S}$  in the film are given by

$$\vec{S} = \frac{1}{2} \operatorname{Re}(\vec{E}_t \times \vec{H}_t^*) = \frac{1}{2} \operatorname{Re}\left(\nu_1^{\prime *} \frac{k^* E_{0t}^2}{\omega \mu_0} \vec{e}_x + i\nu_2^* \frac{-k^* E_{0t}^2}{\omega \mu_0} \vec{e}_y\right),\tag{17}$$

$$\operatorname{Re}(\vec{k}) \cdot \vec{S} = \frac{1}{2} \operatorname{Re}\left(k' \nu_1'^* \frac{k^* E_{0t}^2}{\omega \mu_0}\right).$$
(18)

Equations (17) and (18) indicate that the direction of the microwave energy flow in the film is not in the exactly positive direction of the x axis. The angle between the x axis and the

microwave energy flow direction is  $\theta = \arctan(\frac{\operatorname{Re}(-i\nu_2^*)}{\operatorname{Re}(\nu_1^*)})$  (see figure 2). From the propagation constant  $k = k' - jk'' = \pm \frac{\omega}{c_0\sqrt{\kappa_3\nu_1'}}$  and equation (18), we can see that if the sign of k' is opposite to that of k'', the fields  $\vec{E}$ ,  $\vec{H}$  and wavevector  $\vec{k}$  of the microwave propagating in the film may form an approximately left-handed triplet of vectors; the film becomes a LHM.

#### 3. Numerical simulations

It is noted that for thin soft magnetic films with in-plane uniaxial anisotropy,

$$\frac{1}{\nu_1'} = \frac{\mu_{xx}\mu_{yy} - \mu_{xy}^2}{\mu_{xx}} = \frac{(\omega_x\omega_y + \omega_y\omega_M + \omega_x\omega_M + \omega_M^2) - \omega^2}{(\omega_x\omega_y + \omega_y\omega_M) - \omega^2}.$$
 (19)

For simplicity, assuming the ferromagnetic damping coefficient  $\alpha = 0$ , it is easily seen from equation (19) that  $\frac{1}{\nu_1'}$  becomes negative in the frequency range of  $\omega_{R1}/2\pi < \omega/2\pi < \omega_{R2}/2\pi$  (where  $\omega_{R1} \equiv \sqrt{\omega_x \omega_y + \omega_y \omega_M}$  and  $\omega_{R2} \equiv \sqrt{\omega_x \omega_y + \omega_y \omega_M + \omega_x \omega_M + \omega_M^2}$ ). In certain ferromagnetic films, the anisotropy field  $H_0$  is small compared with the saturation magnetization  $M_s$ ; e.g., for typical permalloy thin films, the anisotropy field  $H_0$  is about 5 mT and the saturation magnetization  $M_s$  is around 1 T [20]. Thus  $\omega_{R1}/2\pi$  hardly exceeds 5.0 GHz,  $\omega_{R2}/2\pi$  may be around 40 GHz and  $\frac{1}{\nu_1'}$  is negative over a broad frequency range from several GHz to about 40 GHz.

Below, numerical calculations are performed for the film with magnetic parameters of  $\omega_H/2\pi = \gamma H_0/2\pi = 0.14 \text{ GHz} (H_0 \approx 5 \text{ mT}), \omega_M/2\pi = \gamma M_s/2\pi = 28 \text{ GHz} (M_s \approx 1 \text{ T})$ and  $\alpha = 0.005$ , which are consistent with the experimental results for Fe–Co–Ta–N and Fe– Co–Al–N films presented in [20]. The obtained  $\mu_{\nu\nu}$  and  $1/\nu'_1$  are shown in figures 3(a) and (b), respectively. It is seen that  $\text{Re}(\mu_{yy})$  is negative in the frequency range from 2.0 to 28.09 GHz, and the frequency range of the negative  $\text{Re}(1/\nu_1')$  is (2.87–39.65 GHz), which is significantly broader than that of the negative  $\operatorname{Re}(\mu_{yy})$  and independent of  $\alpha$  up to a certain large value. For the relative permittivity element  $\varepsilon_{zz}$ , since the electrical resistivity of permalloy thin films is typically around  $\rho \approx 10^{-6} \ \Omega$  m [20], the relaxation time of the metal may be taken as  $\tau \approx 10^{-13}$  s ( $\gamma = 1/\tau \approx 0.1$  eV) [2], and  $\omega_{\rm p} = (\frac{ne^2}{\varepsilon_0 m^*})^{1/2} = (\frac{1}{\varepsilon_0 \rho \tau})^{1/2}$ . We assume  $\omega_{\rm p} = 2\pi \times 100\,000$  GHz and  $\omega_{\rm p}\tau = 100$  [9]; the given  $\varepsilon_{zz}$  is demonstrated in figure 3(c). It is seen that negative  $\operatorname{Re}(\varepsilon_{zz})$  is reached during the frequency range considered in this work. On the basis of the obtained  $1/\nu'_1$  and  $\varepsilon_{zz}$ , the microwave propagation constant k and the inner product  $\operatorname{Re}(k)\vec{e}_x\cdot\vec{S}$  are calculated and plotted in figures 3(d) and (e), respectively. These figures indicate that, in a broad frequency range, the sign of Re(k) is opposite to that of Im(k), and  $\operatorname{Re}(k)\vec{e}_x\cdot\vec{S}<0$ , i.e., the energy flow is in the direction opposite to the microwave vector; thus the film becomes a LHM. It is clear that the bandwidth of this kind of LHM is significantly wider than that of the above mentioned ones [4, 5, 9-11]. Furthermore, the frequency position and range width of negative  $\operatorname{Re}(1/\nu'_1)$  can be altered easily by applying an external magnetic field  $H_{ex}$  according to equation (19); thus the frequency positions and bandwidth of LHM are modified too, which is directly seen from figure 3(f) where taken  $\gamma (H_0 + H_{ex})/2\pi = 2.9$  GHz (i.e.  $H_{\rm ex} \approx 0.1$  T).

Finally, we discuss energy loss of the microwave propagating in the film. The transfer matrix method is applied to calculate the reflection, transmission and absorption. In calculations, the inversion relative permeability element  $1/\nu'_1$  shown in figure 3(b) is adopted. We have studied a lot of films with different plasma frequencies  $\omega_p$  and thicknesses *t*; some typical results are shown in figure 4. It is found from figure 4(a) that the transmission is about -21 dB for the film with  $\omega_p = 2\pi \times 1000000$  GHz and thickness of 100 nm. Decreasing the film



**Figure 3.** The frequency dependence of (a) the relative permeability element of  $\mu_{yy}$ , (b) inversion relative permeability element of  $1/v'_1$ , (c) the relative permittivity element of  $\varepsilon_{zz}$  (=1/ $\kappa_3$ ), (d) the microwavevector constant *k* and (e) the inner product of Re(k) $\vec{e}_x \cdot \vec{S}$ . In (a)–(d), the solid lines refer the real parts of the parameters, and the dotted lines indicate the imaginary parts of the parameters. (f) Inner product of Re(k) $\vec{e}_x \approx 0.1$  T is applied.

thickness also decreases the reflection, and may possibly increase the absorption. An oscillation behaviour of absorption versus film thickness is shown in figures 4(a)-(c). The transmission monotonically increases with decreasing the film thickness. On the other hand, comparing the transmissions of figures 4(e)-(g) with ones of figures 4(a)-(c), we can see that depressing the value of the plasma frequency  $\omega_p$  (i.e., increasing its resistivity) may significantly reduce the energy loss, and lead the microwave to pass through thicker films with high transmission. Therefore, applying thin ferromagnetic film with high resistivity (such as the granular films [9]) may effectively reduce the energy loss of the microwave passing through the film.

# 4. Conclusion

In this paper, we have theoretically studied properties of normally incident E-polarized microwave propagation in the metallic ferromagnetic films with in-plane uniaxial anisotropy.



**Figure 4.** Calculated transmissions (solid lines), reflections (dotted lines) and absorptions (long dashed lines) of the films with different plasma frequencies  $\omega_p$  and thicknesses t, (a)  $\omega_p = 2\pi \times 100\,000$  GHz (the corresponding relative permittivity element of  $\varepsilon_{zz}$  (=1/ $\kappa_3$ ) is shown in figure 3(c)), t = 100 nm, (b)  $\omega_p = 2\pi \times 100\,000$  GHz, t = 50 nm, (c)  $\omega_p = 2\pi \times 100\,000$  GHz, t = 1 nm, (d)  $\omega_p = 2\pi \times 1000$  GHz, t = 200 nm, (e)  $\omega_p = 2\pi \times 1000$  GHz, t = 1000 nm and (f)  $\omega_p = 2\pi \times 1000$  GHz, t = 200 nm. The inversion relative permeability element  $\mu_{yy}$  shown in figure 3(b) is adopted in the calculations.

It is found that, due to gyromagnetism, the metallic ferromagnetic film may be a broadband LHM with the bandwidth and frequency position easily altered by the external magnetic field. Confirmations of the expected properties should motivate further theoretical progress and open a relevant field of applications towards technological purposes.

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